EVALUATION OF THE ACCURACY OF A LOWER CONFIDENCE LIMIT ESTIMATE FOR SERIES SYSTEM RELIABILITY

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THESIS

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by

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ABSTRACT

The purpose of this study is to evaluate the accuracy of a procedure used to compute an estimate of the lower $100(1-\gamma)\%$ confidence limit for reliability of a system of independent components connected in logical series. The procedure takes a Bayesian approach and uses test data on the individual components where the sample sizes may be unequal and no knowledge of the component failure distribution is needed. A computer simulation is used to generate test failure data and to compute estimates for the lower $100(1-\gamma)\%$ confidence limit on system reliability.

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I. INTRODUCTION

The problem of determining economically a lower $100(1-\gamma)\%$ confidence limit on the system reliability of complex and expensive systems has plagued reliability managers for a long time. In 1968 Joseph Bram of the Center for Naval Analyses developed an approximate procedure to attack this problem for systems of independent components connected in logical series.[1] It is the purpose of this study to test the accuracy of Bram's procedure and to determine the limits of its usefulness, especially in applications involving systems of ten or more components. The accuracy measurements are determined through a computer simulation.

In an effort to demonstrate reliability goals on expensive systems, it is usually not economically feasible to test the entire system many times. Bram takes a Bayesian approach using test data, successes and failures, on individual components or subsystems to obtain an estimate for the lower confidence limit on the overall system reliability. The method assumes that the components are independent, and test sample sizes for the various components may be unequal. No knowledge or assumptions about the components' failure rates is needed for the computation, only the test failure data.

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[1] Center for Naval Analyses, O.E.G. Research Contribution No. 79, Confidence Limits for System Reliability, by Joseph Bram, 2 February 1968.

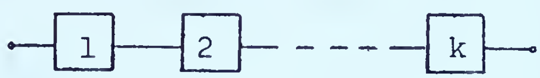
II. RELIABILITY MODEL AND COMPUTATION PROCEDURES

A. THE RELIABILITY MODEL AND COMPUTATION PROCEDURE: METHOD A

1. Summary of the Reliability Model

Consider k independent components, as in Figure 1, connect in logical series with respective individual reliabilities p_1, $p_2, \ldots, p_k$. By the product rule for series systems, the actual overall system reliability, R_s, is given by

$$R_s = \prod_{i=1}^{k} p_i \quad .$$



Component reliability:	p_1	p_2	...	p_k
Number of tests:	n_1	n_2	...	n_k
Number of successes:	s_1	s_2	...	s_k

FIGURE 1.

THE SERIES SYSTEM MODEL

In Bram's Bayesian approach to finding an estimate for the lower confidence bound on system reliability, the p_i's, and hence R_s, are considered to be random variables. If the distribution from which the p_i's were sampled were known, a lower confidence bound could be calculated without

testing. Since this distribution is not known however, the Bayesian's assume a prior distribution which when combined with the test data leads to a posterior distribution, hopefully one that is recognizable and from which confidence limits can easily be computed.

By considering the random variable

$$u = -\ln R_s = -\sum_{i=1}^{k} \ln p_i$$

the procedure approximates the density of u by

$$\phi(u) = \frac{\beta^{\alpha+1}}{\alpha!} u^{\alpha} e^{-\beta u}$$

The parameters α and β will be estimated from test data. This density results from having chosen a prior density for p_i which is questionable on the grounds that it is not normalizable (see prameterization of the prior density, Ref. 1) and leads to a posterior which is also not normalizable. The choice of priors causes difficulty in estimating the parameters of $\phi(u)$ when no failures occur on any of the components, and its effects will be investigated later in this study. However, since this is an approximate method, the development can proceed from the assumption that u does in fact possess the above density and that estimates for α and β can be determined.

If a new random variable v is defined where

$$v = 2\beta u ,$$

then v is distributed "chi square" with $2(\alpha+1)$ degrees of freedom. Then the lower $100(1-\gamma)\%$ confidence limit, $R_{s,L(\gamma)}$, can be found as follows:

$$P[2\beta u < \chi^2_{2(\alpha+1),1-\gamma}] = 1 - \gamma$$

$$P[u < \frac{\chi^2_{2(\alpha+1),1-\gamma}}{2\beta}] = 1 - \gamma$$

$$P[e^{-u} > \exp(- \frac{\chi^2_{2(\alpha+1),1-\gamma}}{2\beta})] = 1 - \gamma$$

$$P[R_s > \exp(- \frac{\chi^2_{2(\alpha+1),1-\gamma}}{2\beta})] = 1 - \gamma$$

From the probability statement above,

$$R_{s,L(\gamma)} = \exp(- \frac{\chi^2_{2(\alpha+1),1-\gamma}}{2\beta})$$

is the desired approximate lower confidence limit. The problem is to find an estimate, $\hat{R}_{s,L(\gamma)}$, for $R_{s,L(\gamma)}$ which will yield values sufficiently accurate for the reliability manager's purposes. Bram's method specifies estimates $\hat{\alpha}$ and $\hat{\beta}$, which are computed from the test data, and substituted into the above expression, to obtain

$$\hat{R}_{s,L(\gamma)} = \exp(- \frac{\chi^2_{2(\hat{\alpha}+1),1-\gamma}}{2\hat{\beta}}) .$$

In the following section Bram's computation procedure for $\hat{\alpha}$ and $\hat{\beta}$ is described.

2. <u>Description of the Computation Procedure</u>

In the following, Bram's procedure for computing $\hat{R}_{s,L(\gamma)}$, will be referred to as method A, and the author's modifications to the original procedure will be referred to as methods B and C.

For descriptive purposes, after testing has been done, consider the k system components to be arranged as follows: 1, 2,..., k_1, k_1+1,..., k, where components 1, ..., k_1 are those which experience no failures, and k_1+1, ..., k are those which experience one or more failures. Let n_i be the number of tests on component i and s_i be the number of successes in the n_i tests. To get $\hat{R}_{s,L(\gamma)}$ by method A proceed as follows:

$$\hat{M}_A = \sum_{i=k_1+1}^{k} \sum_{j=s_i}^{n_i-1} \frac{1}{j} ,$$

$$\hat{V}_A = \sum_{i=k_1+1}^{k} \sum_{j=s_i}^{n_i-1} \frac{1}{j^2} ,$$

$$\hat{\alpha} = \frac{\hat{M}_A^2}{\hat{V}_A} - 1 \quad \text{and} \quad \hat{\beta} = \frac{\hat{M}_A}{\hat{V}_A} , \quad \text{and}$$

$$\hat{R}_{s,L(\gamma)} = \exp\left(- \frac{\chi^2_{2(\hat{\alpha}+1),1-\gamma}}{2\hat{\beta}}\right)$$

$\hat{M}_A$ and $\hat{V}_A$ are estimates of the mean and variance of u under method A. In cases where $(\hat{\alpha}+1)$ is non-integer, interpolation is to be used.

3. Shortcomings of Method A

It should be noted that the double sums do not account for components which experience no failures in testing, and if none of the system's components fails, $\hat{\alpha}$ and $\hat{\beta}$ are not defined: under these circumstances the procedure can not be used as it stands. Herein lies the major obstacle to application of the procedure to highly reliable system testing programs where it is quite possible that no failures will occur. In testing the accuracy of method A, whenever no failures occur, $\hat{R}_{s,L(\gamma)}$ is set equal to unity by the author's choice, since Bram makes no allowance for this possibility.

Secondly, as $\hat{\alpha}$ is defined, it is possible that $2(\hat{\alpha}+1)$, the estimate of the degrees of freedom of the chi square random variable can be less than one. Since a chi square variate must have at least one degree of freedom, some provision must be made to account for this inconsistency. For computation purposes in testing the original procedure of method A and the modifications, whenever $2(\hat{\alpha}+1)$ is less than one, the degrees of freedom are set equal to one by the author's choice. For all fractional values of $2(\hat{\alpha}+1)$ greater than one, linear interpolation between the tabulated integer degrees of freedom are used, as specified in the original procedure.

B. FIRST MODIFICATION TO THE COMPUTATION PROCEDURE: METHOD B

The only way that the true lower confidence limit $R_{s,L(\gamma)}$ can be unity is to have no failures on an infinite number of

tests; this would be impossible to demonstrate. In searching for a more realistic value for $\hat{R}_{s,L(\gamma)}$ when no failures occur, one might be led to believe that $\hat{R}_{s,L(\gamma)}$ should be close to but not equal to one. Due to its discrete character, the procedure of method A will compute one value of $\hat{R}_{s,L(\gamma)}$ which is closest to unity; this will happen when one failure occurs on the component with the greatest number of tests. An approximation of a more accurate estimate of $\hat{R}_{s,L(\gamma)}$ when no failures occur, can be obtained by using one half the original estimates $\hat{M}_A$ and $\hat{V}_A$, where one failure occurs on the component with the most tests. This has the effect of considering a partial failure on the component with the most tests and tends to smooth the discrete distribution of values of $\hat{R}_{s,L(\gamma)}$ near unity.

In summary, computations under method B proceed as follows:

$$\left.\begin{aligned} \hat{M}_B &= \hat{M}_A \\ \hat{V}_B &= \hat{V}_A \end{aligned}\right. \qquad \text{if } k_1 < k, \quad \text{and}$$

$$\left.\begin{aligned} \hat{M}_B &= \frac{1}{2(N_{max} - 1)} \\ \hat{V}_B &= \frac{1}{2(N_{max} - 1)^2} \end{aligned}\right\} \qquad \text{if } k_1 = k.$$

N_{max} is the number of tests on the component with the greatest number of tests.

$$\hat{\alpha} = \frac{\hat{M}_B^2}{\hat{V}_B} - 1 \quad ,$$

$$\hat{\beta} = \frac{\hat{M}_B}{\hat{V}_B} \quad , \quad \text{and}$$

$$\hat{R}_{s,L(\gamma)} = \exp[- \frac{\chi^2_{2(\hat{\alpha}+1),1-\gamma)}}{2\hat{\beta}}]$$

If $2(\hat{\alpha}+1) < 1.0$, then set $2(\hat{\alpha}+1) = 1.0$.

C. SECOND MODIFICATION TO THE COMPUTATION PROCEDURE: METHOD C

In an effort to refine method B so that it might be useful under conditions of very high component reliability and small amounts of testing, a second modification is presented. Method C of computing $\hat{M}$ and $\hat{V}$ is an attempt to further smooth the distribution of $\hat{R}_{s,L(\gamma)}$ and has the effect of adding a partial failure to each component which experiences no failure during testing. For method C computation proceeds as follows:

$$\hat{M}_C = \sum_{i=1}^{k_1} \frac{1}{2\ k_1^2\ (n_i - 1)} + \hat{M}_A$$

$$\hat{V}_C = \sum_{i=1}^{k_1} \frac{1}{2\ k_1^2\ (n_i - 1)^2} + \hat{V}_A$$

$$\hat{\alpha} = \frac{\hat{M}_C^2}{\hat{V}_C} - 1 \quad ,$$

$$\hat{\beta} = \frac{\hat{M}_C}{\hat{V}_C} \quad , \text{ and}$$

$$\hat{R}_{s,L(\gamma)} = \exp[- \frac{\chi^2_{2(\hat{\alpha}+1),1-\gamma}}{2\hat{\beta}}]$$

Again, if $2(\hat{\alpha}+1) < 1.0$, set $2(\hat{\alpha}+1) = 1.0$.

From these computations it is seen that if every component experiences at least one failure, i.e. $k_1 = 0$, then method C reduces to method B. On the other hand, if a system consists of say forty components, and there is limited testing relative to the component unreliability of say ten of the components, one might feel that some adjustment should be made to the overall estimate $\hat{R}_{s,L(\gamma)}$ to account for the lack of testing on those ten components; that is the motivation for this second modification. Furthermore, any method of computing $\hat{R}_{s,L(\gamma)}$ which yields accurate results with less testing than another method will be more desirable economically.

III. SIMULATION PROCEDURE

The purpose of the computer simulation in this study is to demonstrate the accuracy of the procedures described in section II. The simulation considers the system of k components of Figure 1, where the component reliabilities and hence R_s are assumed to be known. For this system tests are simulated by the Monte Carlo technique, generating failure data which is used to compute $\hat{R}_{s,L(\gamma)}$ by each of the three methods A, B, and C. By replicating the procedure 1000 times, 1000 values of $\hat{R}_{s,L(\gamma)}$ are generated which are then arranged in ascending order. If the method of computing $\hat{R}_{s,L(\gamma)}$ is accurate, then $100(1-\gamma)\%$ of the time the true system reliability, R_s, will be greater than or equal to $\hat{R}_{s,L(\gamma)}$. Therefore the $100(1-\gamma)^{th}$ percentile point of the 1000 ordered values of $\hat{R}_{s,L(\gamma)}$, $A_{1-\gamma}$, should be equal to or very close to the value of R_s if the method is accurate. For example, if $\gamma = .10$ then the 90^{th} percentile point or 900^{th} ordered value of $\hat{R}_{s,L(\gamma)}$ should be close to R_s. It is this difference, $|R_s - A_{1-\gamma}|$, that is the primary measure of accuracy of the method of estimation. Another measure of the accuracy is the true level of confidence. The true level of confidence is found by determining the percentage of the values of $\hat{R}_{s,L(\gamma)}$ that are less than or equal to R_s. If the method is accurate the true level of confidence will be very close to $100(1-\gamma)\%$.

One of the main concerns of this study is to determine what minimum amount of testing is necessary to insure that the method will provide accurate results. This minimum amount of testing will be different for each system and will be dependent on the number of components and their reliabilities. For this reason the quantity

$$TT = \sum_{i=1}^{k} n_i(1-p_i)$$

is used as a measure of the amount of testing relative to the system reliability.[2] Low values of TT occur when combinations of high reliability and small numbers of tests occur, and TT increases directly with the number of tests per component and inversely with component reliability. Intuitively one would feel that as TT decreases the accuracy of the computation procedure will decrease since fewer failures will occur, and less information will be obtained. One purpose of the simulation is to determine for what values of TT the various methods meet desired accuracy goals.

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[2]Borsting, Jack R., and Woods, W. Max, A Method for Computing Lower Confidence Limits on System Reliability Using Component Failure Data With Unequal Sample Sizes, Naval Postgraduate School, Monterey, California, June 1968.

IV. RESULTS AND CONCLUSIONS

The analysis of accuracy results are shown in Tables I and II. The cases considered are grouped according to the number of components, system reliability, and value of TT. The component reliabilities, p_i's, are held constant in some cases while the number of tests per component, n_i's, are varied, to yield a range of TT values. In other cases the n_i's are fixed and the p_i's varied. Twenty-three cases of systems with reliabilities ranging from approximately .87 to .95 and consisting of 10, 20, and 40 components are considered.

As expected the tendency of all methods to increase in accuracy as TT increases is substantiated. In addition, the tendency of increased accuracy from methods A, to B, to C is also substantiated. Cases 21, 22, and 23 show clearly that for TT = 10.0 the accuracy of the three methods is essentially the same for all three methods of computation, since with this relatively high amount of testing the methods become identical. These three cases also show that as the number of system components changes, if R_s and TT are held constant, the accuracy of the three methods remains essentially unchanged.

It is up to the user to determine what accuracy is acceptable when employing any procedure to compute $\hat{R}_{s,L(\gamma)}$. For illustrative purposes, if one considers $|R_s - A_{1-\gamma}| < .02$ to be acceptable accuracy, then for a 40 component system

with an overall reliability of approximately .90, the testing program should be such that TT = 6.50 is a minimum and TT > 9.0 would be desirable. As TT decreases below 6.50 there is a marked deterioration in accuracy of all three methods as measured by both $|R_s - A_{1-\gamma}|$ and the true level of confidence versus the desired level of confidence. Another key factor to consider in using any of these methods is that each is sensitive to the individual component test sample size, n_i. If n_i is quite small, say less than 10, for any component regardless of TT, a failure on one of those n_i tests will tend to have a disproportionate effect on $\hat{R}_{s,L(\gamma)}$. Note that in cases 6 through 10 accuracy proves to be not as good as in cases 1 through 5 for k = 20 and corresponding values of TT. This is because the n_i values are generally lower for cases 6 through 10 and in some cases n_i is less than 10.

The results show little difference in accuracy between the 80% (γ = .20) and 90% (γ = .10) confidence levels among the various cases and methods. There is no general tendency for the accuracy to be better at either level although one might expect that accuracy at the 80% confidence level would be better than at the 90% level. For example in case 14A (TT = 6.55) accuracy is better at the 90% level, but in cases 13A (TT = 4.05) and 15A (TT = 9.10) accuracy is better at the 80% level.

These methods are approximations. No analytical justification is given for the modifications suggested. Intuition

has been the guiding factor in development of the methods and the accuracy tests which are based on sound analytical techniques are the basis of acceptance or rejection of the method as a useful tool. Since systems vary widely in their composition, it is not possible to cover them all with a set of specific rules as to when it would be appropriate to use one of the methods for computing $\hat{R}_{s,L(\gamma)}$. However, from these results two general rules follow:

1. For systems of ten to forty independent components TT greater than 10.0 should yield accuracies of $|R_s - A_{1-\gamma}| < .01$ provided that,
2. no component is tested less than 10 times, i.e. $n_i \geq 10$ for all components.

If these two rules are followed it can also be concluded that method B should be employed since there should be no appreciable difference in the accuracies of methods A, B, and C, and B is easier to compute than C. Also B does not suffer from the possibility, remote as it would be if TT were greater than 10.0, of yielding $\hat{R}_{s,L(\gamma)} = 1.0$ as would method A. Any system under consideration can be tested using using the computer simulation procedure, and as TT decreases below 10.0, either because of economic or time constraints in the testing program, this would be recommended.

CASE NO.	K	n_i and p_i	TT	R_s	γ	$A_{1-\gamma}$	$\lvert R_s - A_{1-\gamma}\rvert$	TRUE CONF. LEVEL
1A	20	$n_i=16$, $i=1,\ldots,5$; $n_i=20$, $i=6,\ldots,10$; $n_i=40$, $i=11,\ldots,20$; $p_i=.9975$, $i=1,\ldots,20$.	1.45	.9512	.10	1.0000	.0488	.770
					.20	1.0000	.0488	.563
1B	20	"	1.45	.9512	.10	.9659	.0147	NC*
					.20	.9792	.0280	NC
1C	20	"	1.45	.9512	.10	.9402	.0110	1.000
					.20	.9598	.0088	NC
2A	20	$n_i=6$, $i=1,\ldots,5$; $n_i=30$, $i=6,\ldots,10$; $n_i=100$, $i=11,\ldots,20$. $p_i=.9975$, $i=1,\ldots,20$.	2.95	.9512	.10	.9770	.0258	.668
					.20	.9702	.0190	.514
2B	20	"	2.95	.9512	.10	NC	NC	NC
					.20	NC	NC	NC
2C	20	"	2.95	.9512	.10	.9566	.0054	.869
					.20	.9650	.0138	.563

*NC indicates No Change from corresponding result of previous method.

Summary of Accuracy Results by Case
TABLE I.

CASE NO.	K	n_i and p_i	TT	R_s	γ	$A_{1-\gamma}$	$\lvert R_s - A_{1-\gamma} \rvert$	TRUE CONF. LEVEL
3A	20	$n_i=14$, $i=1,\ldots,5$; $n_i=60$, $i=6,\ldots,10$; $n_i=125$, $i=1,\ldots,20$; $p_i=.9975$, $i=1,\ldots,20$.	4.05	.9512	.10	.9691	.0179	.714
					.20	.9660	.0148	.590
3B	20	"	4.05	.9512	.10	NC	NC	NC
					.20	NC	NC	NC
3C	20	"	4.05	.9512	.10	.9659	.0147	.756
					.20	.9641	.0129	NC
4A	20	$n_i=24$, $i=1,\ldots,5$; $n_i=100$, $i=6,\ldots,10$; $n_i=200$, $i=11,\ldots,20$; $p_i=.9975$, $i=1,\ldots,20$.	6.55	.9512	.10	.9669	.0157	.708
					.20	.9608	.0096	.618
4B	20	"	6.55	.9512	.10	NC	NC	NC
					.20	NC	NC	NC
4C	20	"	6.55	.9512	.10	.9627	.0115	.765
					.20	.9600	.0088	NC

*NC indicates No Change from corresponding result of previous method.

Summary of Accuracy Results by Case
TABLE I.

CASE NO.	K	n_i and p_i	TT	R_s	γ	$A_{1-\gamma}$	$\lvert R_s - A_{1-\gamma} \rvert$	TRUE CONF. LEVEL
5A	20	$n_i=28$, $i=1,\ldots,5$; $n_i=100$, $i=6,\ldots,10$; $n_i=300$, $i=11,\ldots,20$; $p_i=.9975$, $i = 1,\ldots,20$.	9.10	.9512	.10 .20	.9650 .9627	.0138 .0115	.729 .622
5B	20	"	9.10	.9512	.10 .20	NC NC	NC NC	NC NC
5C	20	"	9.10	.9512	.10 .20	.9635 .9671	.0123 .0159	.730 .673
6A	20	$n_i=8$, $i=1,\ldots,5$; $n_i=10$, $i=6,\ldots,10$; $n_i=20$, $i=11,\ldots,20$; $p_i=.995$, $i=1,\ldots,20$.	1.45	.9046	.10 .20	1.0000 1.0000	.0954 .0954	.773 .521
6B	20	"	1.45	.9046	.10 .20	.9312 .9578	.0266 .0532	NC NC
6C	20	"	1.45	.9046	.10 .20	.8808 .9163	.0238 .0117	NC NC

*NC indicates No Change from corresponding result of previous method.

Summary of Accuracy Results by Case
TABLE I.

CASE NO.	K	n_i and p_i	TT	R_s	γ	$A_{1-\gamma}$	$\lvert R_s - A_{1-\gamma} \rvert$	TRUE CONF. LEVEL
7A	20	$n_i=3$, $i=1,\ldots,5$; $n_i=15$, $i=6,\ldots,10$; $n_i=50$, $i=11,\ldots,20$; $p_i=.995$, $i=1,\ldots,20$.	2.95	.9046	.10 .20	.9541 .9406	.0495 .0360	.638 .528
7 B	20	"	2.95	.9046	.10 .20	NC NC	NC NC	NC NC
7 C	20	"	2.95	.9046	.10 .20	.8946 .9289	.0100 .0243	NC .719
8 A	20	$n_i=7$, $i=1,\ldots,5$; $n_i=35$, $i=6,\ldots,10$; $n_i=60$, $i=11,\ldots,20$; $p_i=.995$, $i=1,\ldots,20$.	4.05	.9046	.10 .20	.9362 .9324	.0316 .0278	.697 .575
8 B	20	"	4.05	.9046	.10 .20	NC NC	NC NC	NC NC
8 C	20	"	4.05	.9046	.10 .20	.9295 .9259	.0249 .0213	.713 .588

*NC indicates No Change from corresponding result of previous method.

Summary of Accuracy Results by Case
TABLE I.

CASE NO.	K	n_i and p_i	TT	R_s	γ	$A_{1-\gamma}$	$\lvert R_s - A_{1-\gamma} \rvert$	TRUE CONF. LEVEL
9A	20	n_i=12 , i=1,...,5; n_i=50 , i=6,...,10; n_i=100, i=11,...,20; p_i=.995, i=1,...,20.	6.55	.9046	.10 .20	.9346 .9292	.0300 .0246	.711 .618
9 B	20	"	6.55	.9046	.10 .20	NC NC	NC NC	NC NC
9 C	20	"	6.55	.9046	.10 .20	.9312 .9275	.0266 .0229	.749 .619
10A	20	n_i=14 , i=1,...,5; n_i=50 , i=6,...,10; n_i=150, i=11,...,20; p_i=.995, i=1,...,20.	9.10	.9046	.10 .20	.9315 .9261	.0269 .0215	.723 .624
10B	20	"	9.10	.9046	.10 .20	NC NC	NC NC	NC NC
10C	20	"	9.10	.9046	.10 .20	.9283 .9227	.0237 .0181	.726 .646

*NC indicates No Change from corresponding result of previous method.

Summary of Accuracy Results by Case
TABLE I.

CASE NO.	K	n_i and p_i	TT	R_s	γ	$A_{1-\gamma}$	$\|R_s-A_{1-\gamma}\|$	TRUE CONF. LEVEL
11A	40	$n_i=5$, $i=1,\ldots,15$; $n_i=10$, $i=16,\ldots,30$; $n_i=50$, $i=31,\ldots,40$; $p_i=.998$, $i=1,\ldots,40$.	1.45	.9230	.10 .20	1.0000 .9677	.0770 .0447	.419 .410
11B	40	"	1.45	.9230	.10 .20	.9727 NC	.0497 NC	NC NC
11C	40	"	1.45	.9230	.10 .20	.9450 .9644	.0220 .0414	.772 .610
12A	40	$n_i=5$, $i=1,\ldots,15$; $n_i=10$, $i=16,\ldots,30$; $n_i=100$, $i=31,\ldots,35$; $n_i=150$, $i=36,\ldots,40$; $p_i=.998$, $i=1,\ldots,40$.	2.95	.9230	.10 .20	.9847 .9801	.0617 .0571	.404 .378
12B	40	"	2.95	.9230	.10 .20	NC NC	NC NC	NC NC
12C	40	"	2.95	.9230	.10 .20	.9539 .9650	.0309 .0420	.495 .429

*NC indicates No Change from corresponding result of previous method.

Summary of Accuracy Results by Case
TABLE I.

CASE NO.	K	n_i and p_i	TT	R_s	γ	$A_{1-\gamma}$	$\lvert R_s - A_{1-\gamma} \rvert$	TRUE CONF. LEVEL
13A	40	$n_i=5, i=1,\ldots 5$; $n_i=10, i=6,\ldots,10$; $n_i=20, i=11,\ldots,20$; $n_i=50, i=21,\ldots,30$; $n_i=100, i=31,\ldots,35$; $n_i=150, i=36,\ldots,40$; $p_i=.998$, $i=1,\ldots,40$.	4.05	.9230	.10	.9650	.0420	.649
					.20	.9548	.0318	.570
13B	40	"	4.05	.9230	.10	NC	NC	NC
					.20	NC	NC	NC
13C	40	"	4.05	.9230	.10	.9524	.0294	.701
					.20	.9525	.0295	.594
14A	40	$n_i=5, i=1,\ldots,5$; $n_i=50, i=6,\ldots,20$; $n_i=100, i=21,\ldots,30$; $n_i=150, i=31,\ldots,40$; $p_i=.998$, $i=1,\ldots,40$.	6.55	.9230	.10	.9396	.0166	.785
					.20	.9428	.0198	.625
14B	40	"	6.55	.9230	.10	NC	NC	NC
					.20	NC	NC	NC
14C	40	"	6.55	.9230	.10	.9356	.0126	.825
					.20	.9381	.0151	.639

*NC indicates No Change from corresponding result of previous method.

Summary of Accuracy Results by Case
TABLE I.

CASE NO.	K	n_i and p_i	TT	R_s	γ	$A_{1-\gamma}$	$\lvert R_s - A_{1-\gamma} \rvert$	TRUE CONF. LEVEL
15A	40	n_i= 50, i=1,...,10; n_i=100, i=11,...,20; n_i=150, i=21,...,40; p_i=.998, i=1,...,40.	9.00	.9230	.10 .20	.9294 .9282	.0064 .0052	.861 .757
15B	40	"	9.00	.9230	.10 .20	NC NC	NC NC	NC NC
15C	40	"	9.00	.9230	.10 .20	.9292 .9280	.0062 .0050	.863 NC
16A	40	n_i=15, i=1,...,40 p_i=.999,i=1,...,5;p_i=.998,i=6,...,20 p_i=.997, i=21,...,35; p_i=.990, i=36,...,40.	1.95	.8778	.10 .20	1.0000 .8914	.1222 .0136	.851 .574
16B	40	"	1.95	.8778	.10 .20	.9078 NC	.0300 NC	NC NC
16C	40	"	1.95	.8778	.10 .20	NC .8902	NC .0124	NC NC

*NC indicates No Change from corresponding result of previous method.

Summary of Accuracy Results by Case
TABLE I.

CASE NO.	K	n_i and p_i	TT	R_s	γ	$A_{1-\gamma}$	$\lvert R_s - A_{1-\gamma} \rvert$	TRUE CONF. LEVEL
17 A	40	$n_i=20$, $i=1,\ldots,40$; $p_i=.999, i=1,\ldots,5$; $p_i=.998, i=6,\ldots,20$; $p_i=.997$, $i=21,\ldots,35$; $p_i=.990$, $i=36,\ldots,40$.	2.60	.8778	.10 .20	.8858 .9188	.0080 .0410	.746 .732
17 B	40	"	2.60	.8778	.10 .20	NC NC	NC NC	NC NC
17 C	40	"	2.60	.8778	.10 .20	.8848 .9179	.0070 .0401	.735 .754
18A	40	$n_i=30$, $i=1,\ldots,40$ $p_i=.999, i=1,\ldots,5$; $p_i=.998, i=6,\ldots,20$; $p_i=.997, i=21,\ldots,35$; $p_i=.990, i=36,\ldots,40$	3.90	.8778	.10 .20	.9236 .9017	.0458 .0239	.897 .746
18B	40	"	3.90	.8778	.10 .20	NC NC	NC NC	NC NC
18C	40	"	3.90	.8778	.10 .20	.9229 .9012	.0451 .0234	NC NC

*NC indicates No Change from corresponding result of previous method.

Summary of Accuracy Results by Case
TABLE I.

CASE NO.	K	n_i and p_i	TT	R_s	γ	$A_{1-\gamma}$	$\lvert R_s - A_{1-\gamma}\rvert$	TRUE CONF. LEVEL
19 A	40	$n_i=50$, $i=1,\ldots,40$ $p_i=.999, i=1,\ldots,5; p_i=.998, i=6,\ldots,20;$ $p_i=.997, i=21,\ldots,35;$ $p_i=.990, i=36,\ldots,40.$	6.50	.8778	.10 .20	.8975 .8722	.0197 .0055	.884 .805
19 B	40	"	6.50	.8778	.10 .20	NC NC	NC NC	NC NC
19 C	40	"	6.50	.8778	.10 .20	.8971 .8719	.0193 .0059	NC NC
20 A	40	$n_i=70$, $i=1,\ldots,40$ $p_i=.999, i=1,\ldots,5; p_i=.998, i=6,\ldots,20;$ $p_i=.997, i=21,\ldots,35;$ $p_i=.990, i=36,\ldots,40.$	9.10	.8778	.10 .20	.8745 .8764	.0033 .0014	.902 .820
20 B	40	"	9.10	.8778	.10 .20	NC NC	NC NC	NC NC
20 C	40	"	9.10	.8778	.10 .20	.8743 .8762	.0035 .0016	NC NC

*NC indicates No Change from corresponding result of previous method.

Summary of Accuracy Results by Case
TABLE I.

CASE NO.	K	n_i and p_i	TT	R_s	γ	$A_{1-\gamma}$	$\lvert R_s - A_{1-\gamma}\rvert$	TRUE CONF. LEVEL
21A	10	$n_i=125$, $i=1,\ldots,10$; $p_i=.992$, $i=1,\ldots,10$.	10.0	.9228	.10 .20	.9279 .9290	.0051 .0062	.879 .772
21 B	10	"	10.0	.9228	.10 .20	NC NC	NC NC	NC NC
21 C	10	"	10.0	.9228	.10 .20	.9270 .9280	.0042 .0052	NC NC
22A	20	$n_i=125$, $i=1,\ldots,20$; $p_i=.996$, $i=1,\ldots,20$.	10.0	.9229	.10 .20	.9281 .9290	.0052 .0061	.869 .793
22 B	20	"	10.0	.9229	.10 .20	NC NC	NC NC	NC NC
22 C	20	"	10.0	.9229	.10 .20	.9278 .9281	.0048 .0052	NC NC

*NC indicates No Change from corresponding result of previous method.

Summary of Accuracy Results by Case
TABLE I.

CASE NO.	K	n_i and p_i	TT	R_s	γ	$A_{1-\gamma}$	$\lvert R_s - A_{1-\gamma} \rvert$	TRUE CONF. LEVEL
23A	40	$n_i=125$, $i=1,\ldots,40$; $p_i=.998$, $i=1,\ldots,40$.	10.0	.9230	.10	.9281	.0051	.885
					.20	.9292	.0062	.779
23B	40	"	10.0	.9230	.10	NC	NC	NC
					.20	NC	NC	NC
23C	40	"	10.0	.9230	.10	.9280	.0050	NC
					.20	.9290	.0060	NC
A								
B								
C								

*NC indicates No Change from corresponding result of previous method.

Summary of Accuracy Results by Case
TABLE I.

Case No.	k	TT	R_s	$\lvert R_s - A_{1-\gamma} \rvert$		
				A	B	C
16	40	1.95	.8778	.1222	.0300	NC
6	20	1.45	.9046	.0954	.0266	.0238
11	40	1.45	.9230	.0770	.0497	.0220
1	20	1.45	.9512	.0488	.0147	.0110
17	40	2.60	.8778	.0080	NC	.0070
7	20	2.95	.9046	.0495	NC	.0100
12	40	2.95	.9230	.0617	NC	.0309
2	20	2.95	.9512	.0258	NC	.0054
18	40	3.90	.8778	.0458	NC	.0451
8	20	4.05	.9046	.0316	NC	.0249
13	40	4.05	.9230	.0420	NC	.0294
3	20	4.05	.9512	.0179	NC	.0147
19	40	6.50	.8778	.0197	NC	.0193
9	20	6.55	.9046	.0300	NC	.0266
14	40	6.55	.9230	.0166	NC	.0126
4	20	6.55	.9512	.0157	NC	.0115

Summary of Accuracy Results for $\gamma = .10$
Table II

Case No.	k	TT	R_s	$\lvert R_s - A_{1-\gamma} \rvert$		
				A	B	C
20	40	9.10	.8778	.0033	NC	.0035
10	20	9.10	.9046	.0269	NC	.0237
15	40	9.00	.9230	.0064	NC	.0062
5	20	9.10	.9512	.0138	NC	.0123
21	10	10.0	.9228	.0051	NC	.0042
22	20	10.0	.9229	.0052	NC	.0048
23	40	10.0	.9230	.0051	NC	.0050

Summary of Accuracy Results for $\gamma = .10$
Table II

Case No.	k	TT	R_s	$\|R_s - A_{1-\gamma}\|$ A	B	C
16	40	1.95	.8778	.0136	NC	.0124
6	20	1.45	.9046	.0954	.0532	.0117
11	40	1.45	.9230	.0447	NC	.0414
1	20	1.45	.9512	.0488	.0280	.0088
17	40	2.60	.8778	.0080	NC	.0070
7	20	2.95	.9046	.0360	NC	.0243
12	40	2.95	.9230	.0571	NC	.0420
2	20	2.95	.9512	.0190	NC	.0138
18	40	3.90	.8778	.0239	NC	.0234
8	20	4.05	.9046	.0278	NC	.0213
13	40	4.05	.9230	.0318	NC	.0295
3	20	4.05	.9512	.0148	NC	.0129
19	40	6.50	.8778	.0055	NC	.0059
9	20	6.55	.9046	.0246	NC	.0229
14	40	6.55	.9230	.0198	NC	.0151
4	20	6.55	.9512	.0096	NC	.0088

Summary of Accuracy Results for $\gamma = .20$
Table II

Case No.	k	TT	R_s	$\|R_s - A_{1-\gamma}\|$ A	B	C
20	40	9.10	.8778	.0014	NC	.0016
10	20	9.10	.9046	.0215	NC	.0181
15	40	9.00	.9230	.0052	NC	.0050
5	20	9.10	.9512	.0115	NC	.0159
21	10	10.0	.9228	.0062	NC	.0052
22	20	10.0	.9229	.0061	NC	.0052
23	40	10.0	.9230	.0062	NC	.0060

Summary of Accuracy Results for $\gamma = .20$
Table II

COMPUTER PROGRAM

```
SIMULATION PROGRAM TO EVALUATE THE ACCURACY OF THE LOWER CONFIDENCE
ESTIMATE FOR SERIES SYSTEM RELIABILITY
COMPO = METHOD A OF TEXT
COMP1 = METHOD B OF TEXT
COMP2 = METHOD C OF TEXT
      DIMENSION N(50), P(50), NS(50), GAMMA(5)
      DIMENSION NT(50), NST(50), NFT(50)
      DIMENSION A(5), CONF(5),RSLCL(1000)
      DIMENSION NF(50)
      DIMENSION  PCNTIL(5), CSTABL(100,2)
      DATA N/50*0/, P/50*0.0/, NS/50*0/, GAMMA/5*0.0/
      DATA A/5*0.0/, CONF/5*0.0/, RSLCL/1000*0.0/
      DATA NT/50*0/, NST/50*0/, NFT/50*0/
      DATA NF/50*0/
      DATA  PCNTIL/5*0.0/, CSTABL/200*0.0/
      KOUNT = 0
      ICUT = 0
      NF1 = 0
      ISTART = 65549
      IRN = ISTART
      M = 12493
      KK = 8 * M + 3
      ISTOP = 0
      IFLAG0 = 0
      IFLAG1 = 0
      IFLAG2 = 1
      IF (IFLAG0.EQ.0) GO TO 0020
      ICOMP1 = 0
      ICOMP2 = 0
      IFLAG2 = 0
      GO TO 0050
 0020 IF (IFLAG1.EQ.0) GO TO 0030
      ICOMP1 = 1
      ICOMP2 = 0
      GO TO 0050
 0030 IF (IFLAG2.EQ.0) GO TO 0045
      ICOMP1 = 0
      ICOMP2 = 1
      ISTOP = 1
      GO TO 0050
 0045 WRITE (6,0046)
 0046 FORMAT (12X, 'IFLAG VALUES SCREWED UP')
      STOP
READ INPUT DATA
 0050 READ (5,0100) K, NGAMMA, NREP
 0100 FORMAT (3I5)
      READ (5,0200) (GAMMA(J), J= 1, NGAMMA)
 0200 FORMAT (5F10.5)
```

```
      ILOOP = 0
 0300 ILOOP = ILOOP + 1
      READ (5,0400) N(ILOOP), P(ILOOP)
 0400 FORMAT (1I5, 1F10.5)
      IF(ILOOP.LT.K) GO TO 0300
      DO 0600 I= 1, 100
      READ (5,0500) (CSTABL(I,J), J=1,2)
 0500 FORMAT (2F10.5)
 0600 CONTINUE
      ILOOP = 0
 0700 ILOOP = ILOOP + 1
      PCNTIL(ILOOP) = 1.0 - GAMMA(ILOOP)
      IF(ILOOP.LT.NGAMMA) GO TO 0700
      NPOI = N(1)
ECHO CHECK INPUTS
      IF(NREP.EQ.1000) GO TO 0790
      IF(NREP.EQ.100) GO  TO 0790
      WRITE (6,0780) NREP
 0780 FORMAT(1X,///, 'ERROR, NREP NOT EQUAL TO 1000 OR 100, NREP= 'I10)
      STOP
 0790 WRITE (6,0800) K, NGAMMA, NREP
 0800 FORMAT ('1', 7X, 'INPUT VALUES FOR SIMULATION', //,
     119X, 'K= ', I5, 5X, 'NUMBER OF COMPONENTS IN SERIES',
     2/, 14X, 'NGAMMA= ', I5, 5X, 'NUMBER OF CONFIDENCE LEVELS',
     3/, 16X, 'NREP= ', I5, 5X, 'NUMBER OF REPLICATIONS', /)
      ILOOP = 0
 0900 ILOOP = ILOOP + 1
      WRITE (6,1000) ILOOP, GAMMA(ILOOP)
 1000 FORMAT (12X, 'GAMMA(', I1, ')= ', F5.3, /)
      IF(ILOOP.LT.NGAMMA) GO TO 0900
      WRITE (6, 1100)
 1100 FORMAT (1X, //, 12X, 'COMPONENT        COMPONENT', 5X,
     1'NUMBER OF', 7X, 'NUMBER OF', 7X, 'NUMBER OF',/,13X,'NUMBER',7X,
     2'RELIABILITY   TEST TRIALS   TEST SUCCESSES   TEST FAILURES',//)
      ILOOP = 0
 1200 ILOOP = ILOOP + 1
      WRITE (6,1300) ILOOP, P(ILOOP), N(ILOOP), NS(ILOOP), NF(ILOOP)
 1300 FORMAT (12X, I5, 12X, F6.4, 9X, I6, 10X, I6, 10X,I6,/)
      IF(ILOOP.LT.K) GO TO 1200
      WRITE(6,1400) PCNTIL(1), PCNTIL(2)
 1400 FORMAT (1X, //, 12X, 'SELECTED PERCENTILES FOR THE ',
     1'CHI SQUARED DISTRIBUTION', /, 14X, 'PERCENTILE', 1X, F5.2, 4X,
     2F5.2, /, 18X, 'DF')
      DO 1600 I= 1,100
      WRITE (6,1500) I, (CSTABL(I,J), J=1,2)
 1500 FORMAT (15X, I5,   5X, F6.2, 4X, F6.2)
 1600 CONTINUE
FIND COMPONENT WHICH HAS LARGEST NO. OF TESTS
```

```
      K1 = K - 1
      I = 1
 1610 I1 = I + 1
      IF (N(I).GE.N(I1)) GO TO 1620
      NMAX = N(I1)
      GO TO 1630
 1620 NMAX = N(I)
 1630 IF(I.GE.K1) GO TO 1640
      I = I+ 1
      GO TO 1610
 1640 NMAX1 = NMAX - 1
      XNMAX1 = NMAX1
COMPUTE SERIES SYSTEM RELIABILITY BY PRODUCT ASSUMPTION
      RCOMP = 1.0
      ILOOP = 0
 1700 ILOOP = ILOOP + 1
      RCOMP = RCOMP * P(ILOOP)
      IF(ILOOP.LT.K) GO TO 1700
      WRITE (6,1800)
 1800 FORMAT ('1', 7X, 'SIMULATION OUTPUT', //)
COMPUTE TT
      TT = 0.0
      DO 1900 I= 1,K
      T = N(I) * (1.0 - P(I))
      TT = TT + T
 1900 CONTINUE
BEGIN SIMULATION
 1950 L = 1
 2000 G= GAMMA(L)
      J= 1
GENERATE K VALUES OF NS(I) BY SIMULATING TESTS
 2100 I= 1
      II= 1
 2200 IRN = IRN * KK
      RN = 0.5 + FLOAT(IRN) * 2.328306E-10
      KOUNT = KOUNT + 1
      IF (RN.GT.P(I)) GO TO 2300
      NS(I) = NS(I) + 1
      GO TO 2400
 2300 NF(I) = NF(I) +1
 2400 II = II + 1
      IF (II.LE.N(I)) GO TO 2200
      II = 1
      I = I + 1
      IF (I.LE.K) GO TO 2200
SIMULATION COMPLETE FOR REPLICATION J
COMPUTE M, V, ALPHA, AND BETA
      I = 1
```

```
      EXPU = 0.0
      VARU = 0.0
 2500 INC = 0
      IF (NF(I).EQ.0) GO TO 2700
 2600 S= NS(I) + INC
      EXPU = EXPU + 1.0/S
      VARU = VARU + 1.0 / (S*S)
      INC = INC + 1
      IF (NF(I).GT.INC) GO TO 2600
 2700 I = I + 1
      IF (I.LE.K) GO TO 2500
      IF(ICOMP2.EQ.0) GO TO 2730
COMPUTATION 2
      SUM2 = 0.0
      SUM3 = 0.0
      SUM4 = 0.0
      DO 2720 I= 1,K
      IF (NF(I).NE.0) GO TO 2720
      XNS1 = N(I) - 1
      SUM2 = SUM2 + 1.0/XNS1
      SUM3 = SUM3 + 1.0/(XNS1*XNS1)
      SUM4 = SUM4 + 1.0
 2720 CONTINUE
      D1 = 2.0 * SUM4 * SUM4
      IF (D1.EQ.0.0) GO TO 2730
      EXPU = EXPU + SUM2/D1
      VARU = VARU + SUM3/D1
 2730 IF(VARU.GT.0.0) GO TO 2800
      IF(ICOMP1.EQ.0) GO TO 2750
COMPUTATION 0
      NF1 = NF1 + 1
      EXPU = 1.0 / XNMAX1
      VARU = EXPU * EXPU
      BETA = XNMAX1
      ALPHA = 0.0
      TOBETA = 2.0 * BETA
      DF = 1.0
      GO TO 3200
COMPO WITH NO FAILURES
 2750 BETA = 0.0
      ALPHA = -1.0
      DF = 1.0
      CS = 3.0
      RSLCL(J) = 1.0
      GO TO 3300
 2800 BETA = EXPU / VARU
      TOBETA = 2.0 * BETA
      ALPHA = BETA * EXPU - 1.0
```

```
      DF = TOBETA * EXPU
      IF(DF.LE.100.0) GO TO 2870
      WRITE (6,2860) J, DF
 2860 FORMAT(1X, //, 'IN REP', I5, ', DF GT 100,  DF= ',F10.5,//)
      STOP
 2870 IF(DF.GE.1.0) GO TO 3200
      WRITE(6,2880) J, DF
 2880 FORMAT(1X, //, 12X, 'INREP', I5, ', DF LT 1.0, DF= ', F10.5,
     1'DF SET TO 1.0')
      DF = 1.0
SEARCH CHI SQUARE TABLE FOR CS
 3200 NDF = DF
      NDF1 = NDF + 1
      CS= (CSTABL(NDF1,L) - CSTABL(NDF,L)) * (DF-NDF) + CSTABL(NDF,L)
COMPUTE THE ESTIMATED SYSTEM RELIABILITY LCL RESULTING
FROM THIS SERIES OF SIMULATED TESTS
      RSLCL(J) = 1.0 / (2.7183 ** (CS/TOBETA))
      IF(RSLCL(J).LE.0.1) GO TO 3310
      IF(DF.LT.1.0) DF = 1.0
 3300 IF (IOUT.EQ.0 ) GO TO 3550
OUTPUT FROM REPLICATION
 3310 WRITE (6,3320) J
 3320 FORMAT (1X, ///, 11X, 'RESULTS OF REPLICATION', I5, //)
      WRITE (6,1100)
      ILOOP = 0
 3330 ILOOP = ILOOP + 1
      WRITE (6, 1300) ILOOP, P(ILOOP), N(ILOOP), NS(ILOOP), NF(ILOOP)
      IF (ILOOP.LT.K) GO TO 3330
      WRITE (6,3340) EXPU, VARU, ALPHA, BETA
 3340 FORMAT (1X, //, 16X, 'M= ', F10.5, /, 16X, 'V= ',
     1F10.5, /, 12X, 'ALPHA= ', F10.5, /, 13X, 'BETA= ', F10.5, /)
      WRITE (6,3400)
 3400 FORMAT (1X, /, 5X,'RESULTS USING BRAM APPROXIMATION',/)
      WRITE (6,3500) DF, CS,PCNTIL(L), RSLCL(J)
 3500 FORMAT (15X, 'DF= ', F10.5, /, 7X, 'CHI SQUARE= ', F10.5, //,
     112X, 'ESTIMATED ', F4.2,' LCL FOR SYSTEM RELIABILITY= ',F10.5,/)
 3550 IF (IPOI.EQ.0) GO TO 4100
ZERO OUT NS(I), NF(I), AND ADD N(I) TO NT(I)
 4100 DO 4200 I = 1, K
      NST(I) = NST(I) + NS(I)
      NFT(I) = NFT(I) + NF(I)
      NT(I) = NT(I) + N(I)
      NS(I) = 0
      NF(I) = 0
 4200 CONTINUE
      J = J + 1
      IF (J.LE.NREP) GO TO 2100
PRINT SUMMARY OF RANDOM NUMBER GENERATION AND TEST RESULTS
```

```
      WRITE (6,4300)
 4300 FORMAT ('1', 17X, 'SUMMARY OF SIMULATION RESULTS',//)
      WRITE (6,4400) KOUNT, ISTART,IRN
 4400 FORMAT (1X,//, 12X,'NUMBER OF RANDOM NUMBERS GENERATED= ',
     1I10, /, 12X, 'RANDOM NUMBER GENERATOR SEED= ', I10, /,
     212X, 'LAST RANDOM NUMBER GENERATOR SEED= ', I15, /)
      WRITE (6,1100)
      ILOOP = 0
 4500 ILOOP = ILOOP + 1
      WRITE (6,1300) ILOOP, P(ILOOP), NT(ILOOP), NST(ILOOP), NFT(ILOOP)
      IF (ILOOP.LT.K) GO TO 4500
      WRITE (6,4550) NF1
 4550 FORMAT (/, 12X, 'THERE WERE ' I5, ' REPLICATIONS WITH NO FAILURES
     1', /)
      IF(IOUT.EQ.0) GO TO 5200
      WRITE (6, 4600)
 4600 FORMAT (1X, ///, 12X, 'UNORDERED VALUES OF SYSTEM',
     1' RELIABILITY LCLS, READ LEFT TO RIGHT', /)
 4800 KOUNT2 = 10
      KOUNT3 = 1
 4850 WRITE(6,4900) (RSLCL(J) , J= KOUNT3, KOUNT2)
 4900 FORMAT(1X, 'BRAM', 7X, 10F10.5)
 5100 IF (KOUNT2.EQ.NREP)  GO TO 5200
      KOUNT2 = KOUNT2 + 10
      KOUNT3 = KOUNT3 + 10
      GO TO 4850
ARRANGE THE SYSTEM RELIABILITY LCL VALUES IN ASCENDING ORDER
 5200 NR1 = NREP - 1
      DO 5400 JJ = 1, NR1
      KR = NREP - JJ
      KOUNT1 = 0
      DO 5300 II = 1, KR
      IF (RSLCL(II).LE.RSLCL(II + 1)) GO TO 5300
      KOUNT1 = KOUNT1 + 1
      TEMP = RSLCL(II)
      RSLCL(II) = RSLCL(II + 1)
      RSLCL(II + 1) = TEMP
 5300 CONTINUE
      IF (KOUNT1.EQ.0) GO TO 5450
 5400 CONTINUE
FIND THE VALUE OF RSLCL WHICH IS THE (1-G)TH PERCENTILE
OF ALL THE ORDERED VALUES
 5500 XNREP = NREP
      J1 = PCNTIL(L) * XNREP
      RSLCL1 = RSLCL(J1)
FIND HOW MANY VALUES OF RSLCL(J) ARE .LE. TO RCOMP
FROM THIS FIND THE TRUE LEVEL OF CONFIDENCE
      DO 5600 JJ = 1, NREP
```

```
      IF (RSLCL(JJ).LT.RCOMP) GO TO 5600
      A(L) = JJ-1
      GO TO 5700
 5600 CONTINUE
 5700 CONF(L) = A(L) / XNREP
PRINT OUTPUT
      WRITE (6,5800)
 5800 FORMAT (1X, ///, 12X, 'ORDERED VALUES OF SYSTEM RELIABILITY',
     1 ' LCLS, READ LEFT TO RIGHT', /)
 5820 KOUNT2 = 10
      KOUNT3 = 1
 5830 WRITE (6,4900) (RSLCL(J), J= KOUNT3, KOUNT2)
      IF(IPOI.EQ.0) GO TO 5840
      WRITE (6,5000) (RSLPOI(J), J= KOUNT3, KOUNT2)
 5840 IF(KOUNT2.EQ.NREP)  GO TO 5850
      KOUNT2 = KOUNT2 + 10
      KOUNT3 = KOUNT3 + 10
      GO TO 5830
 5850 WRITE (6,3400)
      WRITE (6,5900) RCOMP
 5900 FORMAT (1X, //, 12X, 'SYSTEM RELIABILITY BY PRODUCT ',
     1'ASSUMPTION RULE= ', F10.5,/)
      WRITE (6,6000) PCNTIL(L), RSLCL1
 6000 FORMAT (12X, 'THE ', F5.2, ' PERCENTILE OF THE ORDERED VALUES ',
     1 'OF SYSTEM RELIABILITY LCL= ', F10.5, /)
      WRITE (6,6100) A(L), CONF(L)
 6100 FORMAT (12X, 'THERE WERE ', F6.2, ' VALUES OF RSLCL ',
     1 'LESS THAN OR EQUAL TO RCOMP', //, 12X,
     2 'THE TRUE LEVEL OF CONFIDENCE IS', F10.5)
      WRITE (6,6200) TT
 6200 FORMAT (12X, 'TT= ', F10.3)
 6260 IF(L.GE.NGAMMA) GO TO 6300
      NF1 = 0
      L = L + 1
      GO TO 2000
 6300 IF (IFLAG2.NE.0) GO TO 6400
PREPARE FOR COMP1
      IRN = ISTART
      NF1 = 0
      ICOMP1 = 1
      ICOMP2 = 0
      IFLAG2 = 1
      GO TO 6500
 6400 IF (ISTOP.NE.0) GO TO 6700
PREPARE FOR COMP2
      IRN = ISTART
      ICOMP1 = 0
      ICOMP2 = 1
```

```
      NF1 = 0
       ISTOP = 1
ZERO OUT TOTALS
 6500 DO 6600 I= 1,K
      NST(I) = 0
      NFT(I) = 0
 6600 CONTINUE
      GO TO 1950
 6700 STOP
      END
INPUT DATA CARDS INCLUDE VALUES FOR K, NO. OF CONFIDENCE LEVELS TO
INVESTIGATE, NO. OF REPLICATIONS, CONFIDENCE LEVELS, COMPONENT
RELIABILITIES, NO. OF TESTS PER COMPONENT, AND A CHI SQUARE DISTRIBUTION
TABLE.
```

LIST OF REFERENCES

1. Center for Naval Analyses, O.E.G. Research Contribution No. 79, Confidence Limits for System Reliability, by Joseph Bram, 2 February 1968.

2. Borsting, Jack R., and Woods, W. Max, A Method for Computing Lower Confidence Limits on System Reliability Using Component Failure Data with Unequal Sample Sizes, Naval Postgraduate School, Monterey, California, June 1968.

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13. ABSTRACT

The purpose of this study is to evaluate the accuracy of a procedure used to compute an estimate of the lower 100(1-γ)% confidence limit for reliability of a system of independent components connected in logical series. The procedure takes a Bayesian approach and uses test data on the individual components where the sample sizes may be unequal and no knowledge of the component failure distribution is needed. A computer simulation is used to generate test failure data and to compute estimates for the lower 100(1-γ)% confidence limit on system reliability.

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